

Calculation of Averages in the Work of Artemas Martin

by

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Some of the research done for this project took place in the Institute in the History of Mathematics and Its Use in Teaching, supported by the National Science Foundation and the Mathematical Association of America, started during the Summer 1995.

Artemas Martin (1835-1918) was a mathematician who spent most of his life in Western New York state and Erie County, in Pennsylvania. He had limited schooling, but studied mathematics as a hobby. He was a prolific writer of mathematics, and started three of the first journals of the American mathematical community: *The Mathematical Monthly* (1859), *The Mathematical Visitor* (1878), and *The Mathematical Magazine* (1882).

His mathematical work was judged by several mathematicians as very good, and in the biography of Martin that appears in the first volume of *The American Mathematical Monthly*, Dr. B. F. Finkel writes "The 'Reprint' contains a large number of his solutions to difficult 'Average' and 'Probability' problems, which are masterpieces of mathematical thought and skill... His style is direct, clear, and elegant. His solutions are neat, accurate and simple."

In this presentation, we will briefly consider the thirteen articles "On Average" that Martin published in *The Wittenberger*, between 1877 and 1879. In the series of articles, various formulas for the calculation of averages in several situations are presented. The first three examples deal with cases of averages of finite collections of numbers. Soon the author passed to examining averages of functions. At the beginning, he used functions of one variable, evolving to two or more variables, and then to vector-valued functions. In one of the articles he presented the calculation of the average of several averages, basically introducing the concept of weighted averages.

One constant in all the solutions presented by Martin is his ability to choose the independent variables so that the average value of the function will be easy to calculate. To reach this goal, he used all kinds of results from geometry and trigonometry. In this way, most of the integrals he obtained can be calculated by hand, even without the use of tables of integration.

One of the first geometric problems appears in the first article "On Average. I," which appeared in the April 1877 issue of *The Wittenberger*, reads as follow: "Required the average area of all right-angled triangles whose hypotenuses are a."

In this case, the variable, x , is used to indicate the length of one of the legs. The other leg is represented by $\sqrt{a^2 - x^2}$. At this point, the area of one of such triangles is represented by

$$A_x = \frac{1}{2} x \sqrt{a^2 - x^2}.$$

The average of such areas is given by

$$A = \frac{1}{a} \int_0^a A_x dx .$$

Martin performs this easy integration to obtain the final numerical result.

A problem which is much more complicated and whose easy solution depends crucially on the appropriate choice of the variable appears in "On Average. II," published in May of the same year. "A point is taken at random in the surface of the circle, and a chord is drawn through it at random, and then a point is taken at random on the chord and another chord drawn to it at right angles to the first. Find the average area of the quadrilateral formed by joining the extremities of the chords."

Martin includes no pictures, and the details are too cumbersome to be presented here.

The next articles included the following problem: "Two equal spheres intersect; find the average of the volume common to both." As an exercise, the reader is asked to generalize the result to spheres with different radii.

In 1878, Martin proposed the following "Find the average distance of one corner of a rectangle from all its points on its surface." This problem involves the calculation of averages of functions of several variables. Several more problems with beautiful geometric constructions are presented in the remaining articles.

Considering the fact that Martin received three honorary degrees from Yale, Rutgers, and Hillsdale College, but never formally attended school, his problem solving techniques are even more impressive.

For more details about Martin and his work, please contact either one of the presenters.