Citibank Models Credit Risk on Hybrid Mortgage Loans in Taiwan

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A new type of hybrid loan in Taiwan consists of a traditional residential mortgage with an attached line of credit. Motivated by declines in Taiwanese property values and unexpected credit losses on all types of loans secured by residential real estate, we developed new statistical models for analyzing the credit risk on traditional mortgages, the hybrid loans, and pure equity lines of credit. Nonstationary Markovian models represent probabilities of transition among different financial states for the three credit instruments. We used logistic and regression models to estimate the losses on defaulted loans and the utilization of credit lines. We calibrated the models with account-level data and integrated them into comprehensive forecasting models that revealed differences in risk profiles among the three types of credit and among different segments of each portfolio.

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To manage assets properly, financial institutions must analyze credit risk on portfolio segments and individual loans. Institutions must consider the potential effects of changes in economic conditions on credit losses and provide for future credit losses in their financial statements. They need to monitor individual accounts and focus their collection efforts where they will be most productive. They must assess changes in financial risk on secured loans when market values of collateral unexpectedly depreciate. Assessment of credit risk is part of due diligence when institutions buy, sell, or package assets as financial derivatives. New credit instruments can have unique characteristics that affect the ways that loans’ risk profiles change over their lives. All these circumstances reinforce Zanakis et al.’s (1986) claim that financial institutions constantly need new analytical tools in their competitive business environments. In this case, Citibank had introduced a new mortgage product (a hybrid consisting of a traditional amortizing mortgage loan coupled with a personal line of credit tied to the same collateral) in a changing foreign business environment. The bank needed a tool for estimating the related credit risk in different market segments and for estimating its exposure to credit losses under different economic scenarios. The credit manager for Asia had previously engaged us to develop analytical models for a variety of mortgage products and leases on mature portfolios of assets in North America. The models had been used over a dozen years in several different institutions to estimate reserves required for credit losses and to help in assessing risk premiums on portfolio segments considered for securitization. While recognizing that we faced a less mature and a more uncertain business environment in Taiwan, we developed and tested models with structures similar to those we had used in North America and applied them to three types of credit instruments in Taiwan (traditional mortgages, home equity lines of credit, and the new hybrid loans). We used the models to gain insight on the comparative risk profiles of the different loan types, to validate forecasts of aggregate credit losses generated by simple extrapolative techniques.
tempered by managerial judgment, to affirm findings from behavioral scoring models independently under development for problem loans, to compare credit risk in different portfolio segments, and to produce statistical evidence of changes in loss experience after issuance of new credit guidelines.

Related Research

Four decades ago, Cyert et al. (1962) introduced the use of a stationary Markov chain to represent the transition of a credit through succeeding stages of delinquency and finally into default. Altman (1989) and Asquith et al. (1989) subsequently showed that the likelihood of default for an active loan changes systematically over the life of an asset. Campbell and Dietrich (1983) revealed how prepayments, delinquencies, and defaults on home mortgages in the United States could be related to the age of the account, the ratio of outstanding principal to the property’s market value (loan-to-value ratio), interest rates, and unemployment rates. Cunningham and Capone (1990) showed differences between fixed-rate and variable mortgages in their tendency toward delinquency, prepayment, and default. Kang and Zenios (1992) described systematic changes in propensity toward prepayment and default as attributed to seasoning and burnout effects. Lawrence et al. (1992) found, in studies of payment patterns on mobile-home mortgages, that borrower delinquency patterns were the dominant indicators of default risk and that information obtained with the loan application diminished in value through time.

Zipkin (1993) used Markov chains in evaluating mortgage-backed securities. Smith and Lawrence (1995) showed how multinomial logit models or, alternatively, normalized probabilities from nonlinear regression models could be used to create nonstationary Markov chains that capture systematic changes in transition probabilities as credits mature. Smith et al. (1996) constructed a comprehensive model for US home mortgages that predicts incidence of prepayment, delinquency, default, and losses on defaulted loans. Rosenberg and Gleit (1994) summarized quantitative methods used in credit management, with particular attention to developing credit scores based on borrower characteristics and credit history. We extended and integrated these various concepts to accommodate the new credit instrument that Citibank issued in Taiwan, where general real estate prices declined in recent years and where the foreclosure process is long compared with practice in North America.

The Modeling Structure

We structured three models for the different types of loans around a nonstationary Markov chain that predicts, at monthly intervals, the likelihoods that loans will be in alternative financial states. For standard mortgages with regular amortization schedules, we defined the alternative financial states of the credit as follows:

1. Current, with payments up to date,
2. Delinquent by one to 29 days,
3. Delinquent by 30 to 59 days,
4. Delinquent by 60 to 89 days,
5. Delinquent by 90 to 119 days,
6. Delinquent by 120 to 149 days,
7. Delinquent by 150 to 179 days,
8. In default (delinquent by 180 or more days) and subject to potential loss, or

These states are the stages of progression toward default upon which analysts base standard roll-rate models of risk for various types of credit. Smith and Lawrence (1995) and Smith et al. (1996) also used these state definitions in more complex models for first and second mortgages.

The hybrid loan with a complementary line of credit calls for a different approach. With a traditional home mortgage, the borrower makes payments according to a fixed schedule and (with stable or rising housing prices) systematically increases the amount of equity held in the property. With the hybrid loan, the borrower has no set pattern for utilization of the complementary credit line. The borrower may gradually draw upon the account up to its credit limit to purchase other goods or services or to meet outstanding credit obligations. Alternatively, he or she may use the credit line in one period and completely pay off the principal in the following period while the mortgage component of the loan amortizes. Thus, the hybrid loan, while secured by real estate, may have some attributes and usage patterns similar
to a mortgage, others similar to consumer credit, and others akin to unsecured credit lines. To model the hybrid loans (with the amortizing components and complementary credit lines), the bank therefore had to integrate the two loans tied to the same collateral into a single record that contained information about the property, terms of the combined loan, characteristics of the neighborhood in which the property was located, characteristics of the borrower at the time of loan application, credit-line utilization, monthly payments, and any associated credit losses (write-offs and recoveries). Drawing on previous experience in modeling independent equity-line loans in the United States, and after analyzing the rates of default and prepayment for different levels of credit utilization on the hybrid loans, we defined nine alternative financial states (with possible transitions as depicted in Figure 1) for the hybrid loans and for pure credit lines without an amortizing component:

1. Unutilized (with an open credit line and zero balance),
2. Low utilization (with a balance over zero percent and up to a designated low percentage of the credit line used),
3. Medium utilization (with greater than low utilization and up to a designated medium percentage of the credit line used),
4. High utilization, current (with greater than medium utilization and up to the credit limit with less than two payments overdue),
5. Overutilized, current (with a balance over the current credit limit and less than two payments overdue),
(6) High utilization, delinquent (with greater than medium utilization and up to the credit limit with two or more payments overdue),
(7) Overutilized, delinquent (with a balance over the credit limit and two or more payments overdue),
(8) Defaulted (delinquent by six or more payment cycles or written off), and
(9) Paid off (account paid in full and closed by the borrower as a good credit).

We did not define separate states for delinquent loans with low-to-medium utilization because the few that existed did not exhibit the same propensity to default as delinquent loans with high utilization of the credit line. This behavior could be expected because borrowers could usually draw further on their credit lines to make payments, thus avoiding further delinquency. Loans can move into the overutilized state if interest is imposed when borrowers are at their credit limits, when borrowers inadvertently draw down beyond their credit limits, or when the credit limit is restricted by the bank for other reasons. Between September 1998 and November 2001, for example, there was a total of 450,302 state transitions (Table 1) with initial states distributed as follows:

—15.89 percent from State 1: unutilized,
—12.65 percent from State 2: low utilization,
—39.63 percent from State 3: medium utilization,
—26.43 percent from State 4: high utilization and current,
—3.87 percent from State 5: overutilized and current,
—1.41 percent from State 6: high utilization and delinquent, and
—0.13 percent from State 7: overutilized and delinquent.

Accounts tend to persist in their current states from one month to the next, or to make transitions to the adjacent states (that is, to change incrementally). Accounts are more likely to return to the state with an unutilized line of credit (reverting to the pure mortgage state) if their utilization is low. Most transitions to the defaulted state occur in accounts that are delinquent with high utilization or overutilized and delinquent. Transitions to the early paid-off state are not so strongly related to the previous state. As expected, loans with unutilized lines of credit have higher than average propensities to shift to the paid-off state—so, however, have loans with high utilization and delinquency. Usually borrowers default because they lack the financial resources to make the payments (inability to pay), although they may occasionally decide to abandon properties with negative net equity, leaving them in the hands of the lender. Prepayment may occur for a variety of reasons. People relocate and sell their homes in the process. Borrowers with financial resources pay off their obligations to avoid interest charges. Others refinance their loans with the same or other institutions when they can obtain more favorable interest rates. The bank occasionally rewrites loans for borrowers in difficulty, with the expectation that they will be able to meet their future obligations.

In aggregate, the model predicts the expected number of accounts in the portfolio that will fall into each of the financial states at the beginning (or end) of each month in the forecasting horizon. It also accumulates the expected value of the associated principal balances. At the beginning of the forecast period, the

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</tr>
</thead>
<tbody>
<tr>
<td>Unutilized</td>
<td>85.62</td>
<td>9.07</td>
<td>2.94</td>
<td>0.56</td>
<td>0.13</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>1.67</td>
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<tr>
<td>Low utilization</td>
<td>9.71</td>
<td>73.63</td>
<td>14.53</td>
<td>0.75</td>
<td>0.16</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>1.22</td>
</tr>
<tr>
<td>Medium utilization</td>
<td>1.22</td>
<td>3.56</td>
<td>82.50</td>
<td>10.72</td>
<td>0.60</td>
<td>0.10</td>
<td>0.00</td>
<td>0.02</td>
<td>1.27</td>
</tr>
<tr>
<td>High utilization, Current</td>
<td>0.39</td>
<td>0.50</td>
<td>13.99</td>
<td>81.35</td>
<td>0.46</td>
<td>1.78</td>
<td>0.01</td>
<td>0.01</td>
<td>1.51</td>
</tr>
<tr>
<td>Over limit, Current</td>
<td>0.59</td>
<td>0.75</td>
<td>4.06</td>
<td>0.41</td>
<td>91.69</td>
<td>0.00</td>
<td>0.96</td>
<td>0.02</td>
<td>1.51</td>
</tr>
<tr>
<td>High utilization, Delinquent</td>
<td>0.06</td>
<td>0.03</td>
<td>2.25</td>
<td>17.43</td>
<td>0.00</td>
<td>69.11</td>
<td>0.28</td>
<td>9.08</td>
<td>1.76</td>
</tr>
<tr>
<td>Over limit, Delinquent</td>
<td>0.00</td>
<td>0.00</td>
<td>0.34</td>
<td>0.34</td>
<td>9.26</td>
<td>0.34</td>
<td>77.02</td>
<td>11.66</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Table 1: Average percentage rates of transition depend on current states.
state of an active account (including its principal balance) is known with certainty. Over the forecasting horizon, the model adjusts the conditional transition probabilities for each individual loan as the corresponding conditional account balances, economic conditions, and values of the security for the loans are forecasted to change. For the hybrid loan, the account balance consists of an amortizing component and a complementary balance on the credit line.

The comprehensive forecasting model has six basic components: (1) a component that estimates the probabilities of transition among the alternative states from one month to the next and thence the likelihoods that the account is in each of the alternative states over the forecasting horizon, (2) a component for estimating credit utilization each month, (3) a component that estimates the likelihood of a loss if the borrower were to default in a given period, (4) a component that estimates the percentage of the loan commitment that the bank will lose if default occurs at the corresponding time, (5) a component for distributing those losses to account for loss recognition over the months following default, and (6) a component that estimates the updated gross appraised value as a percentage of estimated market value at time of default (for use in the rule-based loss accrual). (The mathematical structure of the model and the procedures for calibrating the conditional probabilities are presented in the appendix.)

State Probabilities and Transitions

We need a minimum of 63 equations (for seven possible initial states and nine possible ending states) to determine how the transition probabilities differ among accounts and change through time. In fact, we used 126 estimating equations to take advantage of additional information available at the beginning of the forecasting horizon to refine our estimates of the transition probabilities for the first period. The explanatory variables included economic factors, loan characteristics, collateral characteristics, indicator variables for geographical region to capture differences in real estate markets, and indicator variables for January and February to adjust for the effects of the Chinese New Year. (Other monthly seasonal indicator variables and indicator variables for the number of calendar days in the month were also statistically significant in some equations. Risk managers, however, found the resulting saw-tooth patterns in plots of the transition probabilities to be somewhat distracting. Because the refined seasonality did not affect the final results materially, they preferred a simpler version of the model which incorporated only the January and February seasonal indicators.) The geographic variables allow for differences among regional economies, business practices, and housing markets that the economic variables do not capture. In addition, for the first month in the forecasting horizon, we can consider the specific patterns of historical account states (known in the industry as behavioral score). As a surrogate for the behavioral score, we employ a forecasted next state, which is an index variable between 1 and 8 (representing the states from unutilized through defaulted). We derive the forecasted next state from an exponentially smoothed average with trend (truncated at 0 or 8 if necessary). We include it among the independent variables (along with an interaction term formed with the complement of an indicator of whether the loan is in its first year). This augmented set of explanatory variables gives rise to the other set of 63 equations that we employ for calculating the transition probabilities for the first period of the forecasting horizon.

The specific variables in each group (and their corresponding names in the model’s equations) are the following:

—Current Financial State of Loan
—Implicit variable, as separate sets of transition equations are developed for transitions from each state (with a diminishing number of explanatory variables as delinquency increases and sample sizes decrease accordingly).
—Indicator of whether a credit restriction has been imposed on the account (crrestr).
—Loan Characteristics
—Proportion of loan term in months that has expired (matprp).
—Interest rate on loan (intrate).
—Original loan-to-value ratio for the mortgage loan (origlv).
—Indicator of exceptions to usual guidelines and business practices when issuing credit (gptype).
— Whether the account type is under surveillance and prone to early-default classification (accdef).
— Borrower Characteristics
  — Indicator of whether borrower has college education (colleduc).
  — Indicator of critical credit-qualifying deviation (critdev).
  — Indicator of employment in occupations with historically high delinquency rates (unfavocc).
— Geographic Indicators
  — Central and south (with north as base).
  — Reputational quality of retail market (prim-area).
— Collateral Type
  — Indicator of townhouse or villa, or other type of collateral (thsvilla, othcoll).
  — Combination of collateral type and property location (primthse, primoth, athse).
  — Indicator of whether age of building exceeds 30 years in Taipei City or 20 years elsewhere (oldbldg).
— Economic and Market Variables
  — Estimated loan-to-value (estlv) computed as the ratio of the balance on the amortizing component of the loan, relative to indexed market value of the collateral.
  — Total credit exposure (including mortgage loan and credit line) relative to estimated market value of the collateral (totlv).
  — Difference between interest rate on loan and average interest rate on new loans from the bank (intdif).
  — Regional unemployment rate (unemrate).
— Seasonal Indicators
  — Indicators for January and February to capture effects of the Chinese New Year (Jan, Feb).
— Historical Payment Trends (employed for first-period transitions only)
  — Indicator of whether in first year of loan (frstyr).
  — Exponentially smoothed prediction of financial state score if not in first year of loan (fyscr = (1 − frstyr) * fctscr).

The coefficients of several of the components of the normalized logistic models (for first-period transitions, including predicted state score) are also presented in the appendix (Table A1). Positive coefficients point to increases in the likelihood of transition to the respective states. The number of data points for fitting the models ranged from 583 for loans over the credit limit and delinquent on home mortgage payments to 71,566 for mortgages with unutilized lines of credit. This results in our fitting models with more explanatory variables for the lower-indexed financial states. We use a complementary set of logistic functions (without the behavioral score surrogate) to estimate transition probabilities in subsequent months.

We assess the net impact of changes in the value of an independent variable by altering the characteristics of a loan, its collateral, or economic conditions (through changes in housing appreciation rates or initial market value) accordingly and driving the loan through the forecasting recursion. We report the results of such a simulation in a series of reports and plots of state probabilities, transition probabilities, default rates, and pay-off rates.

As in other studies of credit risk tied to real estate, we found that falling market values resulted in increased delinquencies and defaults on loans and in larger losses following default. Even after accounting for general change in market value, the forecasted states using the credit-score surrogate, the loan characteristics (such as original loan-to-value ratio and whether a credit restriction had been imposed on the account), and characteristics of the collateral (such as town house versus apartment), we found that the geographic indicator variables retain statistical significance and seem therefore to capture other characteristics of the local markets.

In North American studies, we and other analysts found that the likelihood of prepayment is sensitive to the difference in interest rate on the loan and prevailing rates in the marketplace. For this study, the bank officials in Taiwan were unable to produce reliable measures of competitive interest rates on a consistent basis. We therefore used the average rate for new loans issued by the institution itself as the competitive benchmark. The collection of variables that may contain information about incentive and ability to prepay a loan include totlv, crrestr, estlv, intrate, and intdif (defined above). Interest rates on a new loan are affected by prevailing rates at the time of initiation and also by the creditworthiness of the borrower.
A high current interest rate on a loan would provide the borrower with an incentive to refinance when interest rates drop, but with risk-based pricing (that results in higher interest rates for borrowers judged to be higher risks), it may also indicate that the borrower may have trouble qualifying for a new loan. A credit restriction (crest) would similarly indicate a need for the borrower to pay down loans, but the borrower may have difficulty in doing so. These variables are statistically significant as a block and individually in several of the logistic functions involving transition to the paid-off state. It is difficult, however, to employ them individually for stress testing. When, for example, we imposed a one percent drop in future interest rates in simulations of the forecasting model, the projected aggregate prepayment rates for the hybrid loans increased only from 19 to 20 percent over 24 months. We had observed a greater impact of a corresponding change when testing similar scenarios for mortgage loans in North America. The bank’s lack of competitive interest-rate information and short account history with risk-based pricing (causing a confounding of information about incentive to prepay and ability to prepay) precluded our using the model to evaluate risk in connection with future interest rate changes. We therefore made projections for the Taiwanese hybrid loans under a nominally stable interest-rate scenario. We geared economic scenarios to assumptions about the future changes in housing prices (which themselves are admittedly affected by prevailing interest rates).

**Utilization of the Credit Line**

We estimated the degree of utilization of the line of credit for each of the six nonterminal states with nonzero utilization. We employed regression models that use the previous financial state and utilization level as explanatory variables. Utilization within a financial state is higher on average if the borrower made a transition from a high-utilization state and from higher-utilization levels within the previous state. In other words, a separate utilization estimate for the credit line is produced, contingent on the loan’s being in each of the active states. The expected utilization of the credit line at a point of time is computed as the sum of the product of the contingent utilizations and the respective probabilities as determined from the state probabilities. We define the total balance of the outstanding credit as the sum of the mortgage balance and the credit-line balance. We define the total credit exposure $C(t)$ as the sum of the mortgage balance and the larger of the credit-line balance and credit-line commitment.

**Realization of Loss**

The bank may realize a loss when a transition to State 8 occurs. The resulting expected loss in period $t$ is expressed as the product of the probability that a transition occurs to State 8 during period $t$, the probability of nonzero loss on default, and the expected proportion of the total credit exposure (usually credit-line commitment plus balance on the pure mortgage component of the loan) that will materialize as the net credit loss after recoveries.

**Likelihood of Loss on Default**

We estimated the probability of incurring a loss in the event of default by using a logistic function containing variables similar to those used for adjusting the state-transition probabilities. First, we used a regression model to estimate the total outstanding balance on default. Then, we used a derivative of this variable (totlv) in the logistic model for likelihood of loss on default. To allow sufficient time for accrual of losses and recoveries, we calibrated this model using 479 loans that defaulted in the years 1998 through 2001 (with a minimum of 24 months of loss and recovery data through January 2003). Overall, the model was statistically significant at the 0.0001 level. The variable reflecting the combination of the value of the collateral and outstanding loan commitment was marginally significant at the 0.0001 level; the variable reflecting the estimated outstanding loan balance at default relative to the value of the collateral was marginally significant at the 0.05 level. These were the dominant variables in estimating the likelihood of loss on default. Indicators of geographical location and property characteristics added explanatory power in the expected direction. Regional unemployment rate and interest rate on the loan failed to provide predictive power in the expected direction. This may be due to their relation to risk assessment when initiating or rewriting a loan, or to their relation to property values, which are reflected also in the...
Estimating Loss Severity by Allocating the Expected Net Credit Loss

We estimated the expected net credit loss (NCL) as a percentage of loan commitment from data for 373 loans on which the bank incurred losses. Other things being equal, the bank suffered higher percentage losses on loans with high estimated loan-to-value ratios at time of default, with tempering of the estimates according to age of the loan, its interest rate, whether or not the Chinese New Year occurred in the month, and by miscellaneous attributes of collateral. The timing of loss recognition is another issue. The foreclosure and sale of properties can take considerable time. Losses are therefore further distributed among the months following default, in accordance with historical patterns of loss recognition. The net result shows loss rates on a new loan increasing in early years and then decreasing (Figure 2), but to a lesser extent for the hybrid loans than on traditional mortgages.

Estimating Loss by Using the Write-Off Rule

The recognition of losses is, to some extent, a managerial decision and is subject to periodic review. In practice, the bank takes a series of write-downs according to successive appraisals of the value of the collateral. Depending on expectations regarding the supply and demand for housing units, the amount the bank loses on a property is the difference between the outstanding balance on the hybrid loan (mortgage principal plus credit balance) and the expected recovery from sale of the property (a chosen percentage of the updated gross appraised value). The percentage reflects commissions, other costs, and expected sales price relative to appraised value. Because the write-off rules may change, we developed an alternative model for estimating realized loss, which we call the rule-based write-off. At five review points (at default, and at succeeding six-month intervals) the bank realizes the loss according to the applicable rule. To use the write-off rule as an alternative method of forecasting credit losses, one must estimate the updated appraised value of the collateral on the one hand and the total outstanding balance on the loan at the time of default on the other hand. For the former, we use a regression model to estimate the ratio of the updated market value to the indexed market value (the original appraised value adjusted for the regional housing price index) of the home at the time of default. We estimate the logarithm of the updated appraisal ratio as a function of:

- the age of the loan,
- the loan commitment relative to estimated market value,
- the interest rate,
- regional geographic variables,
- whether the loan was issued under new strategic guidelines,
- whether it was a refinancing that was considered to be risky,
- whether the collateral is a town house or villa,
—whether the bank had overlooked a critical deviation on a credit report when issuing the loan,
—whether it is the first updated appraisal following default,
—whether it is the second updated appraisal following default, and
—how many months had passed since default.

We apply the inverse of the logarithm of the updated appraisal ratio to the indexed market value to produce the estimate of the updated appraised value at chosen loss-recognition points (for example, using five points in time, at the time of default and at 180-day intervals thereafter). We parameterize the model in accordance with the projected write-down policy. Specifically, the analyst indicates the number of points in time at which the write-offs are to occur and the percentage of updated appraised value to be assumed as recoverable on disposal of the asset. With the write-off rule, we provide an alternative to the net credit loss allocation and allow for changes in business policy in recognizing losses. In this instance, we developed the model for the updated appraised value from 525 updates on properties securing hybrid loans through 24 months following default.

To estimate the total expected loss associated with the default of loans in period $t$ from the write-off rule, we estimate the loss that will be recognized for individual loans at various review points and accumulate those amounts across accounts to determine the total default risk associated with the portfolio.

The Integrated Model

We integrated these basic constituents (Table 2) into a comprehensive recursive model that considers, for every loan in the portfolio, the probabilities of being in the nine alternative states at monthly intervals over the forecasting horizon, the expected levels of utilization conditioned upon the financial state, the

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<thead>
<tr>
<th>Component</th>
<th>Type of model (Calibration procedure)</th>
<th>Types of variables used</th>
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<tbody>
<tr>
<td>State transition probabilities (126 estimating equations)</td>
<td>Normalized logistics (SAS LOGISTIC) or multinomial logit (SAS CATMOD) or truncated and normalized regression (SAS REG)</td>
<td>Economic variables, loan characteristics, collateral characteristics, borrower characteristics, geographic indicators, seasonal indicators, and historical payment pattern</td>
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<tr>
<td>Expected utilization (six estimating equations for each of the states with a utilized credit line)</td>
<td>Regression (SAS REG)</td>
<td>Previous financial state, previous utilization level</td>
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<tr>
<td>Probability of experiencing a loss on a defaulted loan</td>
<td>Logistic (SAS LOGISTIC)</td>
<td>Economic variables, loan characteristics, collateral characteristics</td>
</tr>
<tr>
<td>Total loss exposure relative to market value of the property on default</td>
<td>Regression (SAS REG)</td>
<td>Loan characteristics, economic variables, geographical characteristics, collateral characteristics</td>
</tr>
<tr>
<td>Percentage of loan exposure lost if loss occurs</td>
<td>Regression (SAS REG)</td>
<td>Loan characteristics, economic variables, geographical characteristics, collateral characteristics</td>
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<tr>
<td>Ratio of updated appraised value to indexed market value at the time of default</td>
<td>Regression (SAS REG)</td>
<td>Loan characteristics, economic variables, geographical characteristics, collateral characteristics, borrower characteristics</td>
</tr>
<tr>
<td>Likelihoods that reappraisal and associated write-downs occur (four estimating equations)</td>
<td>Logistic (SAS LOGISTIC)</td>
<td>Loan characteristics, economic variables, geographical characteristics, collateral characteristics, borrower characteristics</td>
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<tr>
<td>Commitment</td>
<td>Original commitment used to avoid the need to forecast change in commitment</td>
<td>Fixed at original value</td>
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Table 2: The comprehensive model has seven major components with a fixed loan commitment.
likelihood of loss if default occurs, the magnitude of losses on defaulted loans, and expected timing of loss recognition and recovery following default. We aggregate the projections for individual loans to produce corresponding data for the portfolio as a whole or for specified segments. We thus produce monthly projections of the numbers and percentages of accounts in the beginning portfolio that are in each of the nine alternative financial states at the beginning of each month in the forecasting horizon; the percentages of active accounts that are in each of the seven nonterminal states, total expected losses, and the expected loss in dollars in each geographical area. We also produce cumulative summaries for five chosen periods. In a periodic planning cycle, one would recalculate the coefficients of the transition equations using data through the most recent month. One would recalculate the severity model using loss experience for loans that entered the defaulted state at least 24 months ago. This allows enough time for the bank to accumulate the great majority of recoveries from its collection efforts. By running forecasts for different subsets of the portfolio, we can compare their risk profiles.

The resulting risk profile (the product of likelihood of defaulting and severity of default) for a hybrid loan as it ages (Figure 2) is similar to the general pattern reported in research on standard first mortgages in North America (Smith et al. 1996) and similar to the general pattern derived for standard Taiwanese mortgages. Risk increases at first and then decreases as the mortgage component of the loan is amortized and as the collateral increases in value (assuming, in this case, that the value of the collateral appreciates at the rate of three percent per year). This seasoning effect, however, appears to be more muted for the hybrid product than for traditional mortgages. Risk seems to taper off more slowly beyond the fourth year. This behavior is perhaps to be expected, as the borrower can draw on the complementary credit line and increase the loan-to-value ratio at will.

Out-of-Sample Tests of Forecasting Accuracy

We constructed the models based on state transitions from August 1998 through November 2001. To produce an out-of-sample test of the comparative forecasting and discriminating power of the model, we applied the model to accounts active on November 30, 2001 and predicted their status each month through October 2002. We are interested in three types of forecasting accuracy. First, we want to predict the total number of loans in each financial state at the end of each month (with special attention to defaulted loans and the resulting losses). Second, we want to be able to identify the specific loans that are most likely to default. Third, we want to be able to forecast the aggregate net credit losses after recoveries.

The forecasted default rate over the 11-month horizon was predicted to be 2.9 percent, quite close to the actual rate of 3.1 percent. As reported by Smith et al. (1996), much of the explanatory power in the model, particularly when predicting defaults, comes from the financial states of the individual loans at the beginning of the forecasting horizon. To assess the incremental value of other information and modifying the transition probabilities accordingly, we created a version of the forecasting model that employed stationary transition probabilities based on average rates of transition for the same periods used to calibrate the logistic equations. When we used that model, the predicted default rate was 2.5 percent over the 11 months, lower than the 2.9 percent we predicted with the model using nonstationary transition probabilities and lower again than the actual default rates of 3.1 percent (Table 3).

Accounts in the test period were paid off at a dramatically faster rate than forecasted (39 percent actual versus eight percent forecasted). This turned out to be caused by a boom in refinancing caused by a drop in interest rates and more aggressive lending on the part of competing institutions during 2002–2003. The bank had deliberately followed a cautious strategy amidst the refinancing boom. While we predicted defaults more accurately with the additional information, ironically, we predicted prepayments more accurately by using average transition rates.

The forecasted distributions of the states of loans that remain active (not shown) were quite close to actual values, although aggregate delinquencies (on highly utilized and overutilized accounts) crept up at a higher rate than projected. This rate may arise partly because such accounts were less attractive risks for
refinancing and partly because of calibration errors for a new financial product. The median age of the loan for the transitions used to calibrate the models was 26 months.

We looked at how well the model identifies the loans that are most likely to default by sorting the loans in declining order of their forecasted probabilities of being in the defaulted state at the end of October 2002 and comparing the predicted versus actual default rates for groups of 1,000 accounts. Both the predicted and actual default rates show systematic decreases, revealing quite good discriminating power for defaults (Table 4). When we performed a similar analysis for loans paid off, it was evident that we were not so successful in discriminating among loans likely to be prepaid (Table 4). Although both predicted and actual pay-off rates are 50 percent higher for the groups of loans considered most likely to be paid off than for the groups of loans considered least likely to be paid off, the model’s discriminating power for loans likely to be prepaid is weak in the intermediate categories.

The bank was also interested in whether the model was in agreement with projections from a separate behavioral scoring model devised specifically for ranking potential problem loans according to their likelihood of default over a six-month period. The bank was developing behavioral scores for loans with at least a minor delinquency to identify those that most require the attention of the collections department. The scores were based on many of the variables used in our model and on additional information about the status of other financial accounts. We compared the discriminating data for defaults for (1) our model, (2) our model using average transition probabilities instead of the logistic equations, and (3) the bank’s behavioral score (Table 5). The discriminating ability of the hybrid mortgage model and the bank’s behavioral scoring model were very similar for the subset of loans for which the bank had produced behavioral scores. Both the hybrid mortgage model and the behavioral scoring model were significantly better than using average transition probabilities on the current state of the loan.

<table>
<thead>
<tr>
<th>Months ahead</th>
<th>Using nonstationary transition probabilities (126 equations)</th>
<th>Using stationary transition probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecasted percent default</td>
<td>Actual percent default</td>
</tr>
<tr>
<td>1</td>
<td>0.31</td>
<td>0.39</td>
</tr>
<tr>
<td>2</td>
<td>0.71</td>
<td>0.85</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>1.20</td>
</tr>
<tr>
<td>4</td>
<td>1.34</td>
<td>1.49</td>
</tr>
<tr>
<td>5</td>
<td>1.60</td>
<td>1.70</td>
</tr>
<tr>
<td>6</td>
<td>1.84</td>
<td>1.98</td>
</tr>
<tr>
<td>7</td>
<td>2.06</td>
<td>2.26</td>
</tr>
<tr>
<td>8</td>
<td>2.27</td>
<td>2.45</td>
</tr>
<tr>
<td>9</td>
<td>2.47</td>
<td>2.67</td>
</tr>
<tr>
<td>10</td>
<td>2.67</td>
<td>2.87</td>
</tr>
<tr>
<td>11</td>
<td>2.86</td>
<td>3.05</td>
</tr>
</tbody>
</table>

Table 3: Forecasts are more accurate with nonstationary probabilities.

<table>
<thead>
<tr>
<th>Cumulative number of accounts</th>
<th>Using nonstationary transition probabilities</th>
<th>Using stationary transition probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecasted marginal default percent</td>
<td>Actual marginal default percent</td>
<td>Forecasted marginal prepayment percent</td>
</tr>
<tr>
<td>1,000</td>
<td>11.9</td>
<td>17.0</td>
</tr>
<tr>
<td>2,000</td>
<td>2.7</td>
<td>3.3</td>
</tr>
<tr>
<td>3,000</td>
<td>2.7</td>
<td>2.3</td>
</tr>
<tr>
<td>4,000</td>
<td>1.3</td>
<td>0.6</td>
</tr>
<tr>
<td>5,000</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>6,000</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>7,000</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>8,000</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>8,742</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 4: Discriminating power was strong for defaults and weak for prepayments.

<table>
<thead>
<tr>
<th>Cumulative number of accounts</th>
<th>Using nonstationary transition probabilities</th>
<th>Using stationary transition probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model predicted default percent</td>
<td>Actual default percent</td>
<td>Average roll-rate predicted default percent</td>
</tr>
<tr>
<td>1,000</td>
<td>3.9</td>
<td>3.4</td>
</tr>
<tr>
<td>2,000</td>
<td>1.5</td>
<td>1.1</td>
</tr>
<tr>
<td>3,000</td>
<td>0.9</td>
<td>1.0</td>
</tr>
<tr>
<td>4,000</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>4,966</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5: Discriminating ability is superior to that of average roll-rate extrapolations and very similar to that of bank’s behavioral score for problem loans.
Finally, we considered the projections of credit loss made with the two approaches. Banks’ aggressiveness in recognizing loss and timing recoveries on recognized losses can be highly variable because of business practice, economic conditions, and legal constraints. The NCL allocation resulted in a cumulative forecasted loss that was 15 percent higher than that based on the loss rule over the 11-month period. Considering the great amount of uncertainty involved in projecting aggregate losses in this business environment and the considerable difference in the two approaches, this degree of accord in total projected loss was quite satisfying. The timing of loss recognition under the two approaches was, however, quite different, with an apparent delay in recognizing losses for the NCL allocation. The allocation by loss rule was better in this regard. Only 31 percent (the percentage derived from historical data) of the NCL was recognized in the first month following default under NCL allocation. The bank, in recent years, has apparently been more prompt in recognizing potential loss. We should bring the allocation of NCL following default into alignment with more recent practice by adjusting the percentages used to distribute the losses among periods following default. We left them intact for this paper, however, to avoid contaminating our out-of-sample tests.

Discussion and Conclusion

Forecasting with statistical models using account-level data can improve accuracy and enrich insight. It allows the production of contingency forecasts and assessment of risks for portfolio segments and helps to reveal business dynamics. Plots of projected state probabilities and losses on default reveal different risk profiles for traditional mortgages, hybrid loans, and home-equity lines of credit as an account ages.

Banks typically use a number of models for different aspects of credit management, often separately analyzing default risk in aggregate and prepayment rates in aggregate and using separate behavioral scoring models to identify specific loans that are most likely to default. Our integrated model makes a major contribution in accounting for interactions among these individual elements while providing perspective on changes in portfolio performance. The models can be augmented with indicator (dummy) variables to help managers to assess the impact of interventive strategies, such as changes in credit policy (for issuing new loans or restructuring problem loans) or marketing strategy. In this case, for example, we augmented the model for regular mortgages to include indicator variables at times in 2002 when the bank issued new credit-policy bulletins that would affect collections or credit-granting decisions. The bank credit-strategy variables were highly statistically significant, and assumptions about their continuing effect had a substantial impact on forecasts generated from January 2003 on.

The forecasts made by our approach are an informative complement to forecasts derived with other methods. By using average transition probabilities in place of those adjusted with the logistic functions, one can generate results comparable to those that would be obtained from aggregate roll-rate models and compare them for base-line purposes. Then, one can use the comprehensive model to estimate the likely change in portfolio performance as market values change and the portfolio seasons.

To facilitate monthly updating, we embedded the model in a forecasting system that automatically regenerates equations and prints them to a file that is imported to the forecasting program. The users can alter the parameters that serve to define low and medium levels of account utilization and select different levels of statistical significance for retaining variables in the model. They can experiment with different model configurations and include new explanatory variables, such as indicator variables that coincide with documented changes in business practice.

Our model, in this instance, was quite effective at estimating default risk and identifying the specific loans that were most likely to default. Its discriminating ability for problem loans was comparable to other behavioral scoring techniques developed separately with a much narrower focus. It revealed statistically significant changes in portfolio performance following changes in business practices. Hidden factors in the Asian business environment wreaked havoc, however, with its estimation of prepayment rates. Lore had it that competitive banks were much more eager than Citibank to capture refinancing business in the
midst of low interest rates at that time. Some institutions might also have been qualifying borrowers under governmental programs for new home owners even though they were refinancing existing homes. We would need further research and market intelligence to deal with the prepayment issue.

Appendix. Mathematical Representation of the Model for Hybrid Mortgage Loans

To represent the model mathematically, we define

\[ P_j(t) = \text{probability that the loan is in state } j \text{ at the beginning of period } t, \]

\[ p_{kj}(t) = \text{probability that a loan that is in state } k \text{ at the beginning of period } t \text{ will make a transition to state } j \text{ for the beginning of period } t+1, \]

\[ U_j(t) = \text{expected proportion of credit line utilized if the loan is in state } j \text{ at the beginning of period } t. \]

Then, allowing for nonstationary transitional probabilities, we express the likelihood that a loan is in state \( j \) at time \( t+1 \) as a Markov chain.

Approximation of Multinomial Logistic Equations for the Transition Probabilities

A natural approach for estimating the transition probabilities is to employ a multinomial logistic structure and to use maximum likelihood procedures to produce seven sets of eight logistic functions of the form

\[ \log(p_{kj}(t)/p_{ko}(t)) = f_{kj}(X(t)) \]

for computing the probabilities of transition to ending states \( j = 1, 2, \ldots, 9 \). The logistic expressions involve \( X(t) \), a vector of explanatory variables (some continuous, others categorical) that are applicable to period \( t \).

We found that an approximation to the multinomial logistic equations produces virtually identical results and is much more efficient computationally. We produce seven sets of nine logistics functions of the form

\[ \log(p_{kj}(t)/p_{ko}(t)) = f_{kj}(X(t)), \]

and then normalize the resulting probabilities for each set so that they add to 1.0. In this case, \( p_{kj}(t) \) refers to the likelihood of being in any state other than state \( j \). Some transitions have probabilities very near zero, and we set them to zero (or very small fixed values) accordingly if we have insufficient data points for constructing an estimating equation. State 8 (defaulted) and State 9 (paid off) are absorbing states, thus rendering \( p_{8k} = 1 \) and \( p_{99} = 1 \).

State Probabilities

\[ P_j(t+1) = \sum_{k=1}^{7} p_{kj}(t) * p_{ko}(t) \quad \text{for } j = 1, 2, \ldots, 9. \]  \( (1) \)

Utilization of the Credit Line

We designate the expected utilization of the credit line at the beginning of period \( t \) as \( U(t) \) and compute it as

\[ U(t) = \sum_{j=1}^{7} P_j(t) * U_j(t). \]  \( (2) \)

Expected Credit Loss by Allocation of NCL

The expected loss due to a default occurring on the account in period \( t \) is expressed as

\[ E(\text{LOSS}_t) = [P_8(t+1) - P_8(t)] * \Pi_t * (\rho_t/100) * C(t), \]  \( (3a) \)

where \( E(\text{LOSS}_t) \) is the expected loss in period \( t \); \( \Pi_t \) is the likelihood of incurring a loss if the loan goes into default during period \( t \); \( \rho_t \) is the percent of the credit exposure \( C(t) \) that the bank can expect to lose if a loss occurs from a default in period \( t \). Loss associated with default in month \( t \) but recognized \( f \) months later (in accordance with historical patterns of loss recognition) is

\[ E(\text{DLOSS}_{t+f}) = E(\text{LOSS}_t) * \delta_{tf}, \]  \( (3b) \)

where \( \delta_{tf} \) is the net proportion of credit loss (write-off less recovery) that is to be realized \( f \) months following default. The sum of the \( \delta_{tf} \) values is 1.0. Values at the end of the recovery interval may, however, be negative, as recoveries dominate write-offs.

Realization of Loss by Write-Down Rules

At five review points (at default and at succeeding six-month intervals), the bank realizes loss by write-down rules that are expressed mathematically as

\[ E(\text{DLOSS}_{t+f}) = [P_8(t+1) - P_8(t)] \]

\[ * \text{Max}(0, C(t) - r_f * \text{UPDAPP}_{t+f}) \]

\[ - \sum \text{DLOSS}, \]  \( (4) \)
Table A1: Coefficients of selected logistic functions for transition probabilities are consistent with expected impact of factors.

*Note.* Significance level: \(a = 0.1\), \(b = 0.01\), \(c = 0.001\), \(d = 0.0001\.)
where \( r_f \) is the proportion of the updated appraised value \( f \) months following default that is expected to be recovered when sold and \( \sum DLOSS \) is the cumulative write-off that would have been expected to be realized on the loan prior to period \( t + f \) if default had occurred in period \( t \).

References